

An examination of Libyan and Turkish Students' Thinking Levels Through Their Quadratic Patterns Generalization According to SOLO Taxonomy*

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Abstract

This study aimed to an examination of Libyan and Turkish students' thinking levels of quadratic patterns generalization according to SOLO taxonomy. The study model was a hybrid design with predominantly qualitative design. In this study, the working group was consisted of 112 from Libyan students at three high school in the city of Tripoli in Libya and 129 Turkish students at three high school in the city of Kastamonu in Turkey. The study was undertaken in the 2018-2019 academic year. Overall, the results showed that most Turkish and Libyan students who participated in the study mostly their level on the Uni-structural level, the Multi structural level and the Relational structure level in quadratic patterns generalization, However, some of them were able to reach successful in moving Advanced thinking levels in this area. Despite of these, there Libyan and Turkish students left the problems blank and there were students at the pre-structural level. In order for students to achieve a higher level of thinking in understanding and quadratic patterns generalization. Supporting students to develop their thinking level by solving problems and getting them involved in different activities that are aimed at promoting their skills and improving their level thinking in quadratic patterns generalization. In addition, mathematics teachers should create opportunities for students to relate their mathematical ideas and concepts to enable the students to promote their ability in thinking, as well support students' thinking levels in moving from a design-based learning environment that encourages "recalling facts" to one that encourages "analysis and synthesis".

*This study was derived from the first author's doctoral dissertation titled "Investigation of Turkish and Libyan Students' Algebraic Thinking Skills in Quadratic Equations with one Unknown According to SOLO Taxonomy," submitted to Kastamonu University Institute Science

Introduction

Mathematics is one of the most important sciences in this world, its value and importance have gradually increased throughout history. Mathematics occupies a prominent place in the educational curricula of various countries world (Polat, 2010), and is considered fundamental at all educational levels, its importance is particularly pronounced at the primary level, as it provides students with the foundational mathematical knowledge necessary for continuing their mathematical studies in later educational stages (Isik & Tarim, 2009; Yildirim, 2006).

Mathematics and its various branches play a major role in developing students' thinking skills. Thinking is the basis of the learning process. Developing thinking skills is primary educational requirement, as it leads to a deeper understanding of the knowledge content students acquire. Algebraic thinking is one of the most important types of thinking associated with mathematics, and it is considered a Wayport for students' thinking skills, given its effective contribution to solving mathematical problems, it has an important role in mathematics, and it is a way of seeing the real world (NCTM, 1992).

Different researchers have tried to reveal components of algebraic thinking. For Example, Dindayal (2003) sees algebraic thinking includes three skills the use of symbols and algebraic relationships, the use of multiple representations, the use of patterns and generalizations. Çelik (2007) sees, algebraic thinking consists of three basic skills: using symbols and algebraic relations, utilizing multiple representations, and formulating generalizations (Çelik, 2007). As for Çelik (2007) and Bağdat (2013) also emphasizes that algebraic thinking consists of the ability to use symbols and algebraic relations, benefit from multiple representations and formulate generalizations. Through literature, generalization is an important indicator of algebraic thinking, and it is an essential skill in algebraic thinking. Algebraic thinking is a process that serves the aim of generalization (Kaput, 2008). Pattern and generalisation are both thought to be fundamental to mathematics and Patterns are a key step in the formation of generalization (Hargreaves, Shorrocks-Taylor & Threlfall, 1998).

In our daily life, because there are a system and pattern in many matters in our life, patterns may be described as structures which human encounter in every area of daily life area (Palabıyık & Akkuş-İspir, 2011). In mathematics, patterns are one of its most important topics, and it's the heart and essence of mathematics. (Zazkis & Liljedahl, 2002). Patterns are any predictable regularity (Mulligan & Mitchelmore, 2009). Patterns can be classified as numerical or non-numerical (Smith, 1997). Patterns can be classified as linear and quadratic (Stacey, 1989). as well as patterns can be classified into numerical patterns, pictorial patterns, repeating patterns, and linear and quadratic patterns (Zazkis & Liljedahl, 2002)

Many researchers have sought to study the stages or levels of development of pattern recognition ability. There are studies that show students do not have difficulty with generalization of patterns, for example, in a study conducted by Cathcart et al. (2003) asserted children can learn to see relationships and generalize. In a study conducted by Lannin (2005), which aimed to understand students' thinking and their ability to generalizations, the results showed that students can easily generalize the pattern. In study was conducted by Lee and Freiman (2006), results showed researchers found that children could discover patterns more easily at an early age. In a study made by Amit & Neria (2008), it was aimed to focus on the generalization methods used by students in solving linear and non-linear pattern problems. Results showed that students can easily generalize patterns. In a study made by Akkan (2013), it was aimed to determine strategies when dealing with problems related to linear and quadratic patterns. As a result of the study, it showed students' efficiencies of generalizing patterns and the variety of their strategies.

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However, there are some other studies that have indicated many of the difficulties students encounter in patterns generalization. For example, in a study made by Çayır and Akyüz (2015), it was aimed to determine the pattern generalization strategies of 9th grade students, a result of the study, students had difficulty in finding the generalization rule algebraically. As for study conducted by Andini and Suryadi (2017), As a result of the study, most students could not understand pattern generalization problems. As for study made by Yeşildere and Akkoç (2011), it was aimed to determine the strategies which a total of pre-service teachers used to teach how to find the rule of patterns. As a result of the study, it was determined that the pre-service teachers had some difficulties related to patterns.

It is clear in these previous studies that patterns were among the subjects in algebra which some students had difficulty comprehending and there were already some difficulties related to the concept of pattern, and how to perceive the pattern algebraically (Lee, 1996). For students to become creative in understanding patterns and perceive an algebraically useful pattern. Overcoming all these difficulties, it is necessary to know students' thinking levels of pattern understanding and to give recommendations and suggestions that mitigate and reduce these difficulties. Therefore, this study aims to an examination of Libyan and Turkish Students' Thinking Levels Through their Generalization in Quadratic Patterns according to SOLO Taxonomy. This study is like some studies that identified the level of students' thinking and their knowledge of patterns, as well as in the tools used and their use of SOLO Taxonomy as in mathematics at different levels in the literature as a study (Lam & Foong, 1996; Pegg & Coady, 1993; Bağdat, 2013; ÇELİK, 2007; Elazzabi & Kaçar, 2020). The current study differs from previous studies in that it examines the thinking levels of students of two countries in quadratic patterns generalization according to SOLO Taxonomy. Therefore, being in line with the stated aim, answers to the following questions were sought in the study:

1. What are Libyan and Turkish students' thinking levels Through their quadratic patterns' generalization according to SOLO Taxonomy?
2. Is there a significant difference between the thinking levels of Turkish and Libyan students to quadratic patterns generalization?

Method

The Study Design

As it is descriptive in nature, The case study design was chosen as the most suitable for the nature of the current research objectives, which is one of the qualitative research methods, which allows to determine the attitudes, successes, and ideas of the group of participants (Miriam, 1991; Stake, 1994). The problem-solving thinking levels that the students applied to the quadratic patterns' generalization problems were analyzed according to the answers obtained by the developed measurement tool. SOLO Taxonomy was used to examine the students' thinking levels. The qualitative data obtained from the measurement tool prepared in Arabic and Turkish were analyzed.

Participants

In this study, the participants was consisted of 112 Libyan students at three high school in the city of Tripoli in Libya and 129 Turkish students at three high school in the city of Kastamonu in Turkey. The study was undertaken in the 2018-2019 academic year. As the schools were willing to open their

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doors to carry out the study, convenient sampling was the primary choice for the working group selection (Patton, 1987). Since these students were in the high grade in high schools and they were about to finish from high study, the working group had already studied quadratic patterns and they had the knowledge of quadratic patterns in order to solve the questions asked in this study.

Data Collection Tools

In the study, a four-problems test regarding quadratic patterns generalization was given to the students as data collecting tool, and In addition to Benefit some from the literature as (Ontario, 2013) were also consulted in developing this test. The data of the study were obtained through this test. The test was conducted in the 2018 –2019 academic year. Test is shown in the finding section. Each of the problems in the test are designed to reflect the students' level of thinking in quadratic patterns generalization. Academics' views in mathematics education were taken in Faculty of Education, Kastamonu University with research experience in algebraic thinking and pattern generalization reviewed the instrument to determine whether the questions were appropriate and valid in terms of language, level, and content. For the reliability of data obtained from the test, compatibility between the encoders was taken into consideration. After the students' answers were encoded independently by two supervisors, the student's answers were encoded independently by two supervisors, the Cohen's Kappa concordance coefficient was calculated. Results of Cohen's kappa coefficient were 0.87. Therefore, this result is enough for measuring the consistency of the encoder analysis.

Data Analysis

In this study, according to the descriptive research method the students' responses were analysed according to SOLO taxonomy. The students' responses examined in detail and their thinking levels regarding generalization in quadratic pattern according to SOLO taxonomy. The thinking levels of SOLO Taxonomy and how these assessments were undertaken are explained in below (Biggs & Collis 1991):

Pre-structural Level (PSL): This is the lowest level of SOLO taxonomy on which students cannot understand the problem. Misinterpretation of the problem. Unrelated answers.

Uni-structural Level (USL): In this level, students understanding of the problem is limited. Students focus on one aspect of the problem. Since students focus on only one aspect of the problem, so the answers are limited and unenough.

Multi-Structural Level (MSL): In this level, students can establish a more complex relation. Students use several aspects of the problem without comprehending the relations between them, so students cannot connect these aspects. This means there is no relational connection between students answers.

Relational-Structural Level (RSL): In this level, students can generalize the relation. Students give a rule or formula for the relation. Students understand all the aspects of the problem in relation to the answer. Therefore, they can connect these aspects in this level.

Abstracted-Structural Level (ASL): This is the highest level of SOLO taxonomy on which students can thinking and generalize beyond the data. Students create new things. Students go beyond the problem to develop a new strategy to reach a solution.

Students' responses to the test were classified with the help of a numerical scale depending on the SOLO taxonomy (Mooney, 2002; Rider, 2004). The answers were coded as:

"1" For Pre-structural level (PSL),

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"2" For answer at the Uni- Structure level (USL),

"3" For an answer at the Multi- Structure level (MSL),

"4" For answer at the relational Structure level (RSL).

In addition, when students no answered for questions (Blank), the symbol is "0".

In this way, thinking levels of Libyan and Turkish students regarding quadratic pattern generalization according to SOLO taxonomy were determined. Obtained results were interpreted by conducting percentage, arithmetic average, graphic shapes, Mann Whitney U-Test was used as an alternative test of T-test.

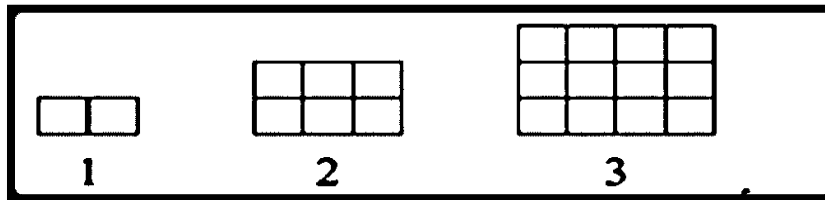
Finding

In this study, The responses of the Libyan and Turkish students were analyzed according to SOLO taxonomy, then the findings obtained were presented in graphs with examples from students. At least one sample was presented from both countries and student answers will be displayed at the highest level students have reached in every problem. They were carefully selected to represent other In addition, data obtained from study were analysed with SPSS computer program.

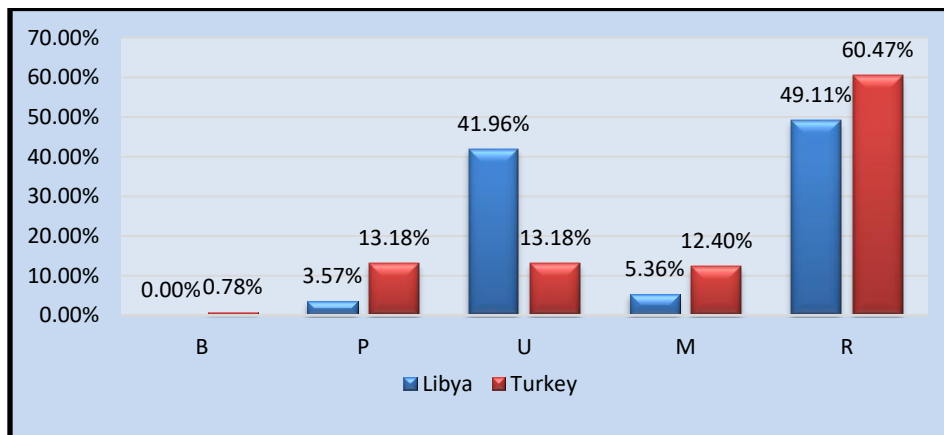
Findings and Comments Related to the First Research Question

The findings of the analysis of the students' responses to the first problem.

problem 1: Find the rule that relates the pattern number to the number of blocks and identify how the pattern is growing.



The analysis of the answers given to the 1st problem according to the SOLO taxonomy is given in the Graph 1.



Graph 1. The Analysis Of The Answers Given To The 1st Problem According To The SOLO Taxonomy

As is seen in the graph1, the findings indicate that a majority of the Turkish and Libyan students participating in the study can demonstrated relational-structural level thinking in solving the first problem. As for the Libyan students, 49.11 % of are at the relational-structural level (RSL), and For the Turkish students, 60.47% of are at the relational-structural level (RSL). In this problem, it means that students thinking levels in regarding generalization quadratic patterns are high. In addition, Turkish students' level was higher than the Libyan student's level. Some of the answers given by the Libyan and Turkish students at the relational -structural level are given below:

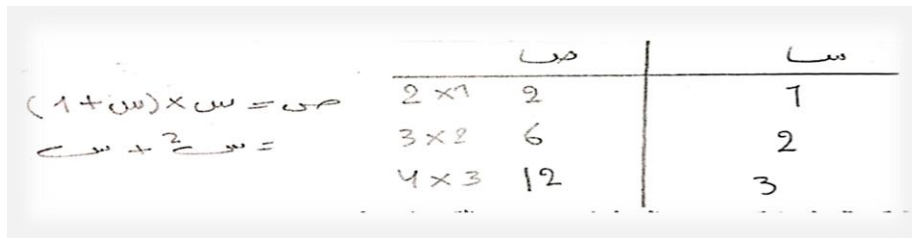


Figure 1. a Libyan students' sample answers to first problem

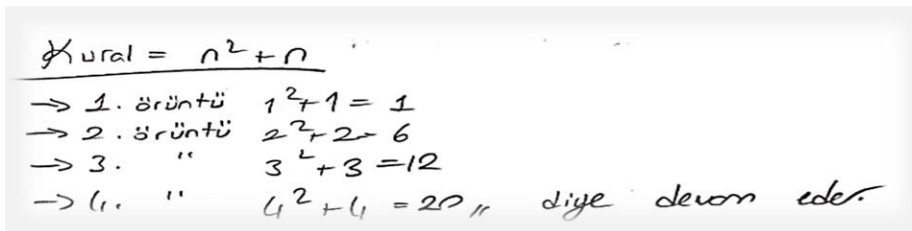


Figure 2. a Turkish students' sample answers to first problem

As for figures 1 and 2 students were able to provide a correct answer to this question. Students have the necessary knowledge and skills to present a solution using the right method, students took on a good step in the solution, students were able to solve the problem by using appropriate steps, and students can formulate a pattern rule. They could generalize the relation and put a rule, they could find the rule that relates the pattern number to the number of blocks, and they could identify how the pattern is growing. For this reason, the students’ answers accepted to be at RSL.

The findings of the analysis of the students’ responses to the second problem.

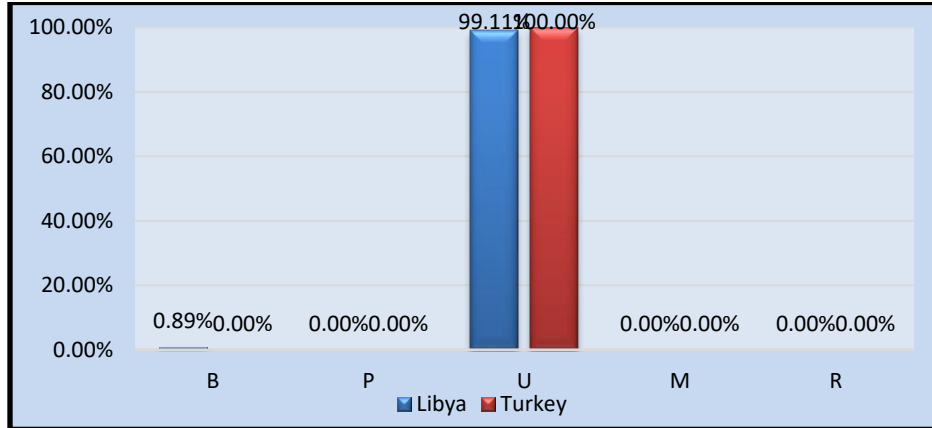
Problem 2: The following table shows the side length and area of a square. Side length of square = x.

Side length	1	2	3
The area	1	4	9	64

- (a) Find the missing value on the table.
- (b) Do you see any patterns in the table, if so? Describe them.
- (c) If there is pattern, what is the formation rule of the numbers in the pattern?

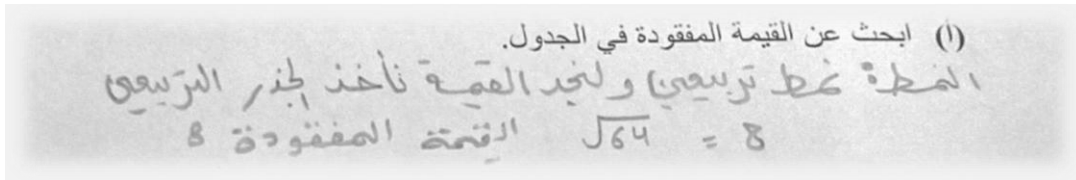
Here present the findings of the analysis of the students’ responses to the second problem, which is in three parts.

Problem 2 - Part a: The analysis of the answers given to the 2nd problem - part (a) according to the SOLO taxonomy.



Graph 2. The analysis of the answers given to the 2nd problem according to the SOLO taxonomy: Part a

As is seen in the graph 2, findings indicate that a large proportion of the Libyan and Turkish students participating in the study can succeed in answer to part (a), the most common level for students in both countries was the Uni-structural level. Where the Libyan students, 99.11 % of was at the uni-structural level (USL), and For the Turkish students, 100% of are at the uni-structural level (USL). In addition, Turkish students' level was higher than the Libyan students' level. Some of the answers given by the Libyan and Turkish students at the uni-structural level are given below:



[Taking the square root of 64, the missing value is 8.]

Figure 3. A Libyan students' sample answers to second problem – Part a

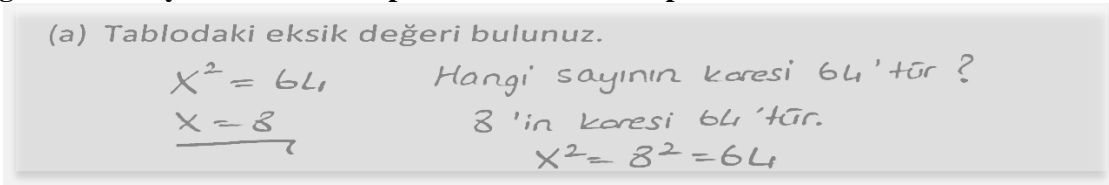
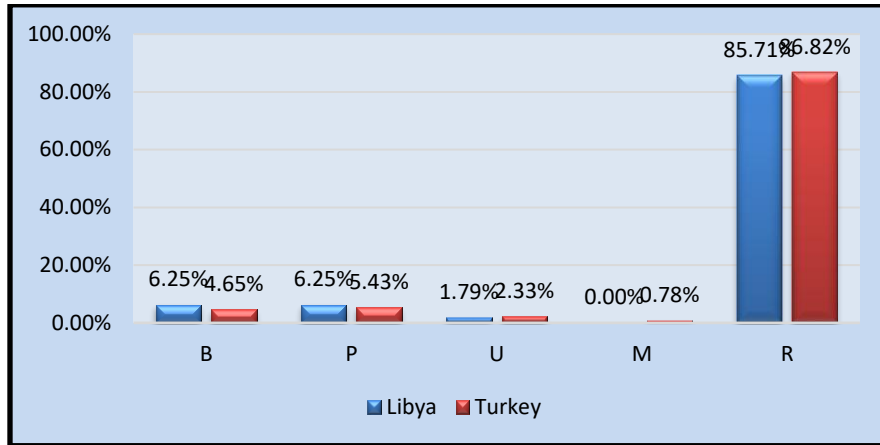


Figure 4. A Turkish students' sample answers to second problem – Part a

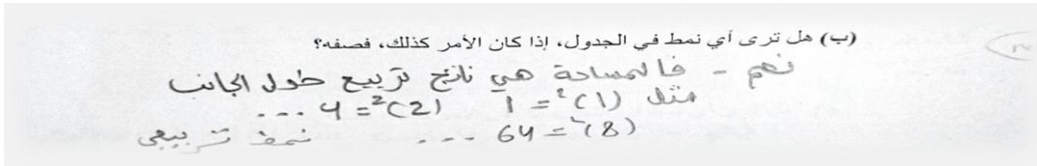
As shown in Figures 3 and 4, these two students could provide a correct answer to this problem, which suggests that they had the necessary knowledge and skills to present a solution using the right method. The students took on a good step for the solution and they can complete the solution, this indicates students' understanding of quadratic patterns. For this reason, the students' answers were accepted to be at USL in part (a).

Problem 2 - Part b: The analysis of the answers given to the 2nd problem - part (b) according to the SOLO taxonomy.



Graph 3. The Analysis of The Answers Given to the 2nd Problem According to The SOLO Taxonomy: Part B

Here, it was observed that a large proportion of the Turkish and Libyan students participating in the study can succeed in advancing toward advanced thinking levels. As for the Libyan students, 85.71 % of are at the relational-structural level (RSL), and For the Turkish students, 86.82% of are at the relational-structural level (RSL). It means that students thinking levels in regarding the formulation of quadratic patterns are high and a large percentage of the Libyan and Turkish students were able to knowledge the pattern and its description. Some of the answers given by the Libyan and Turkish students at the relational-structural level are given below:



[area is squaring the side length, and the pattern is a quadratic pattern]

Figure 5. A Libyan students' sample answers to second problem – Part b

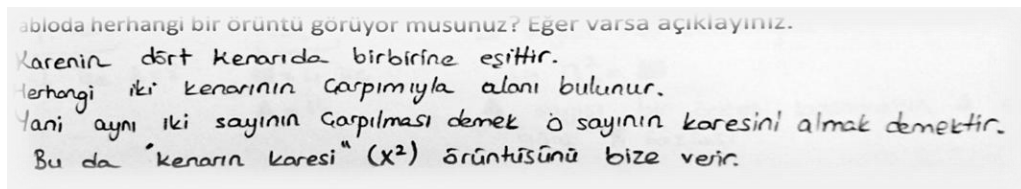
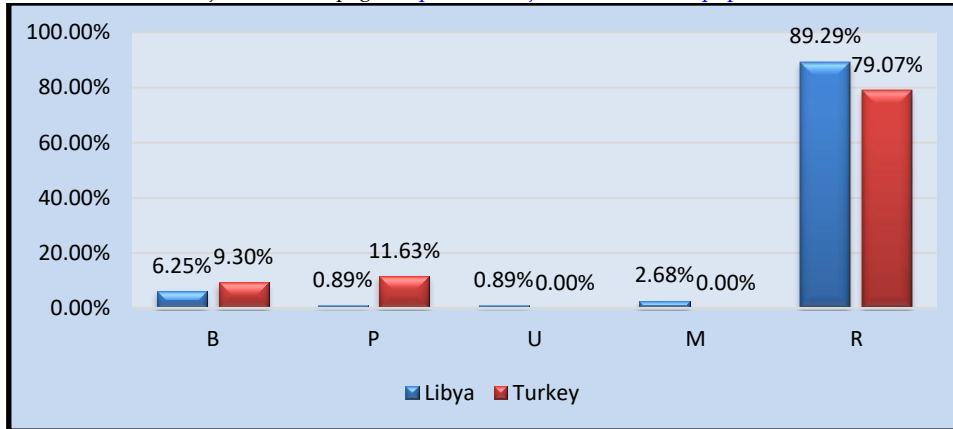


Figure 6. A Turkish students' sample answers to second problem – Part b

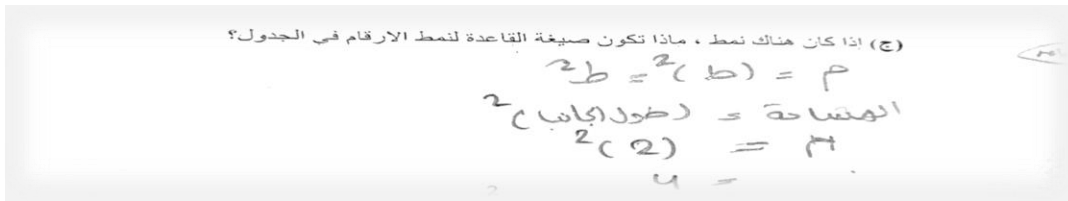
As shown in Figures 5 and 6, these two students could provide a correct answer to this problem, which suggests that they had the necessary knowledge and skills to present a solution using the right method. The students took on a good step for the solution and they were able to knowledge the pattern and its description, this indicates students' understanding of quadratic patterns . For this reason, the students' answers were accepted to be at RSL in part (b).

Problem 2 - Part c: The analysis of the answers given to the 2nd problem - Part (c) according to the SOLO taxonomy.



Graph 4. An analysis of the answers given to the 2nd problem according to the SOLO taxonomy: Part c

As is seen in the graph4, it was observed that a large proportion of the Turkish and Libyan students participating in the study can succeed in advancing toward advanced thinking levels. As for the Libyan students, 89.29 % of are at the relational-structural level (RSL), and For the Turkish students, 79.07 % of are at the relational-structural level (RSL). It means that students thinking levels in regarding the formulation of quadratic patterns are high and a large percentage of the Libyan and Turkish students were able to knowledge the pattern and find the formation rule of the pattern. Despite this, Libyan students' level was higher than the Turkish students' level. Some of the answers given by the Libyan and Turkish students at the relational-structural level are given below:



[The area is equal to the square of the side length, $m = (L)^2$, $L =$ side length]

Figure 7. A Libyan students' sample answers to second problem – Part c

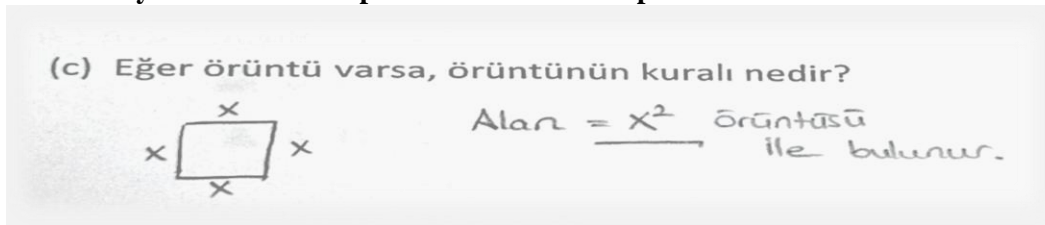


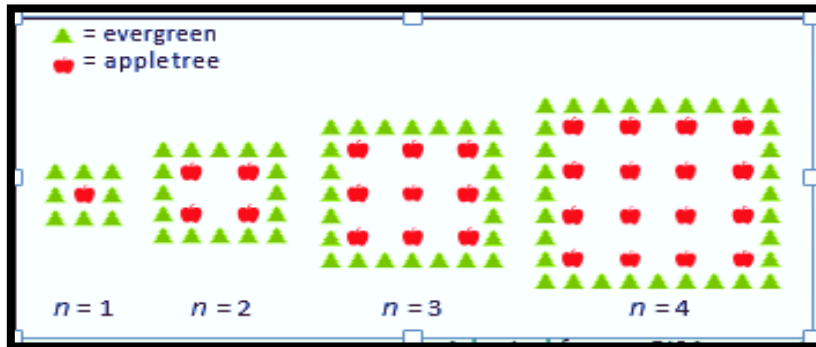
Figure 8. A Turkish students' sample answers to second problem – Part c

In the answers given in figures 7 and 8, where students were able to provide a correct answer. Students have the necessary knowledge and skills to present a solution using the right method. Students took a good step in the solution, they could find the formation rule of the pattern, and they could generalize the relation and put a rule of the pattern. For this reason, the students' answers accepted to be at RSL.

The findings of the analysis of the students' responses to the third problem.

Problem 3: Ali, his neighbors who farm apples. They plant their apple trees in square patterns in each orchard. To protect the trees from the wind, they plant evergreens all around the orchard the diagram

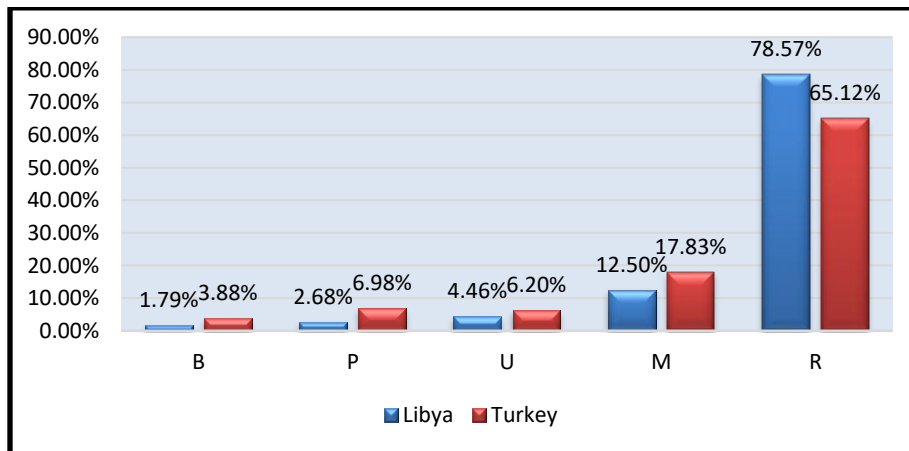
to the below illustrates the pattern of apple trees and evergreens for any number (n) of rows of apple trees.



- Do you see any patterns in the diagram? If so, describe them.
- How many of each type of tree will be there when n = 6?
- When does the number of apple trees equal the number of evergreens? Justify your response.
- How does the growth of the number of apple trees compare with the growth of the number of evergreens?

Here present the results of the analysis of the students' responses to the third problem, which is in four parts.

Problem 3 - Part a: The analysis of the answers given to the 3rd problem - part (a) according to the SOLO taxonomy.



Graph 5. The analysis of the answers given to the 3rd problem according to the SOLO taxonomy: Part a

As is seen in the Graph5, findings indicate that a large proportion of the Turkish and Libyan students participating in the study can succeed in advancing toward advanced thinking levels. As for the Libyan students, 78.57 % of are at the relational-structural level (RSL), and For the Turkish students, 65.12% of are at the relational-structural level (RSL). In this problem, it means that students' thinking levels in regarding the formulation of quadratic patterns are high. In addition, Libyan students' level

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was higher than the Turkish students' level especially at the relational-structural level. Some of the answers given by the Libyan and Turkish students the relational structural level is given below:

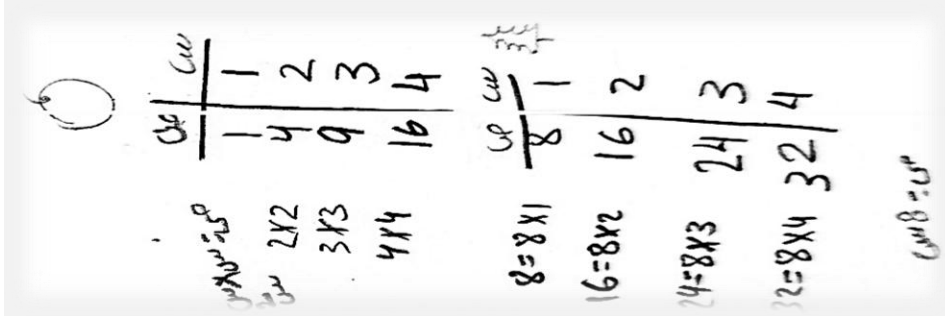


Figure 9. a Libyan students' sample answers to third

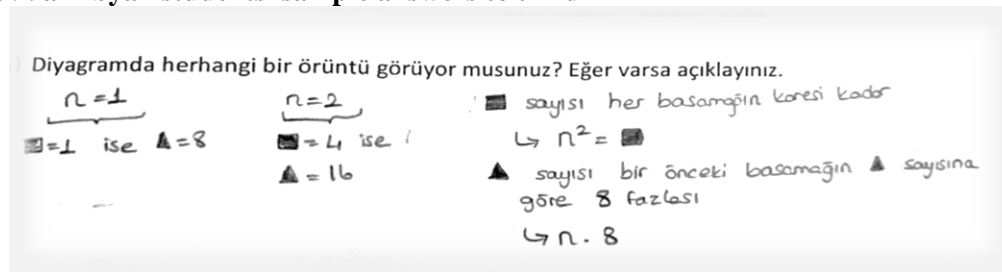
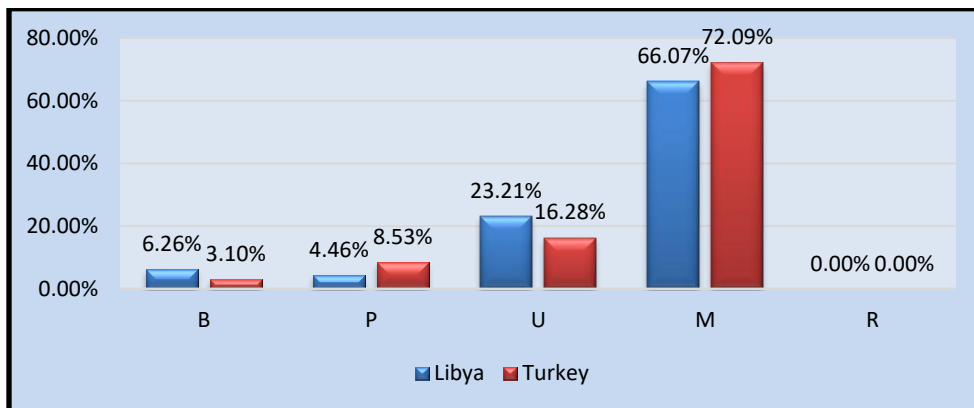


Figure 10. a Turkish students' sample answers to third problem – Part a

As for figures 9 and 10 students were able to provide a correct answer to this problem. Students have the necessary knowledge and skills to present a solution using the right method, students took on a good step in the solution, students were able to solve the problem by using appropriate steps. Where students can see the pattern in the diagram, description of pattern and they could identify how the pattern is growing, and also they could generalize the relation and put a rule. For this reason, the students' answers accepted to be at RSL.

Problem 3 - Part b: The analysis of the answers given to the 3rd problem - part (b) according to the SOLO taxonomy.

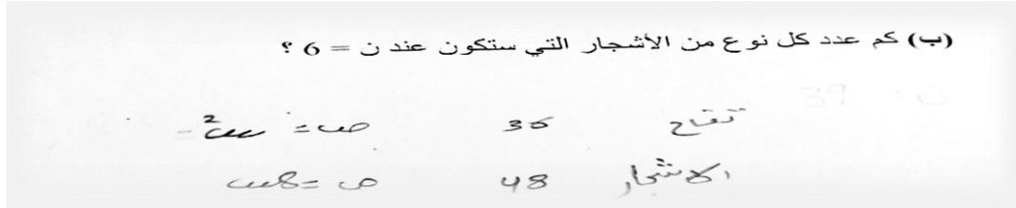


Graph 6. The analysis of the answers given to the 3rd problem according to the SOLO taxonomy: Part b

As is seen in the Graph6, findings indicate approximately 66.07% of the Libyan students tried to solve the problem at multi-structural, 23.21% are at the uni- structural level (USL), and 4.46% were at the pre-structural level (PSL). For the Turkish students, 72.09 % of are at the multi-structural level (MSL),

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16.28% are at the uni- structural level (USL), and 8.53 % ware at the pre-structural level (PSL). In this question, most levels of the students were multi-structural level. Some examples from solutions of students are as follows:



[Apple = $36/y = x^2$ but trees = $48 / y = 8 x$]

Figure 11. a Libyan students' sample answers to third problem – Part b

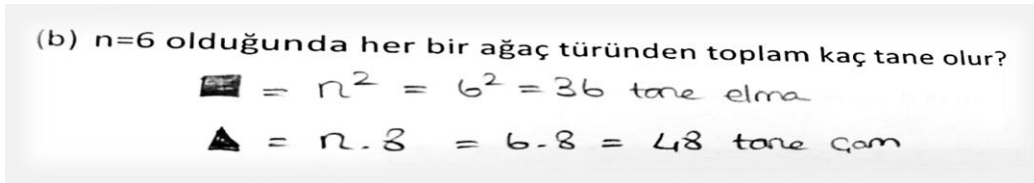
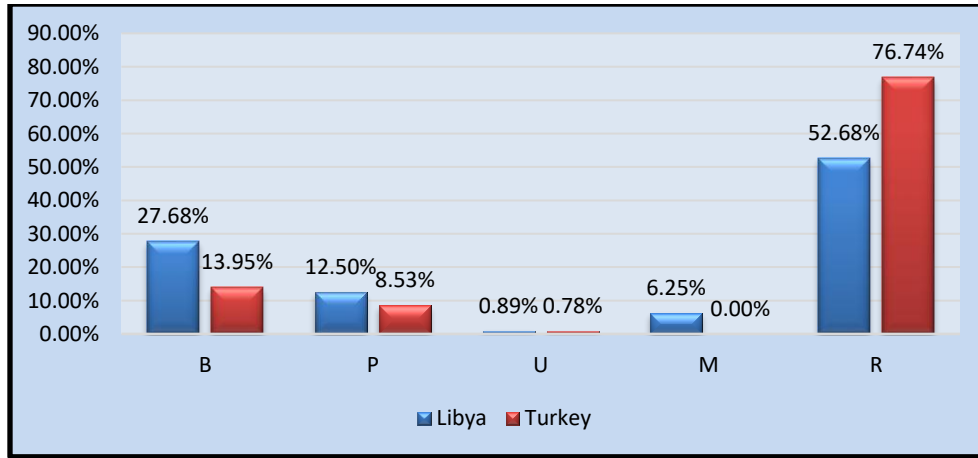


Figure 12. a Turkish students' sample answers to third problem – Part b

As shown in Figures 11 and 12, these two students could provide a correct answer to this problem. The students took on a good step for the solution and the students' answer indicates their knowledge of how the pattern grew , so students were able to know the number of tree species at $n = 6$. For this reason, the students' answers were accepted to be at MSL in part (b).

Problem 3 - Part c: The analysis of the answers given to the 3rd problem - Part (c) according to the SOLO taxonomy.



Graph 7. The analysis of the answers given to the 3rd problem according to the SOLO taxonomy: Part c

As is seen in the Graph7, the findings indicate that a majority of the Turkish and Libyan students participating in the study can demonstrated relational-structural level thinking in solving the problem, suggesting a relatively high level of understanding of quadratic pattern generalization. The following samples give examples of answers that are acceptable at RSL:

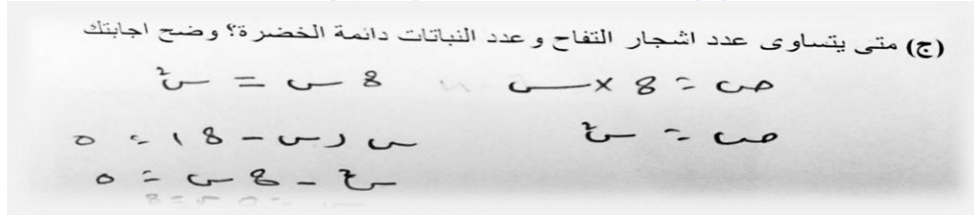


Figure 13. a Libyan students' sample answers to third problem – Part c

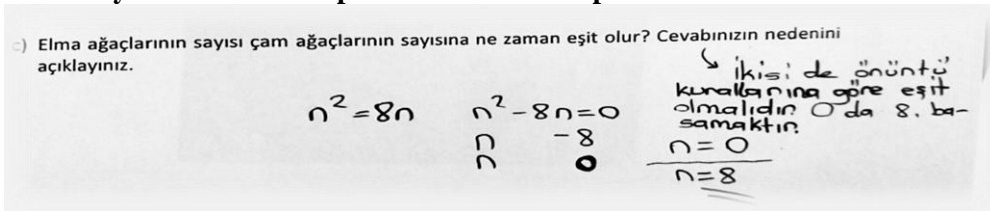
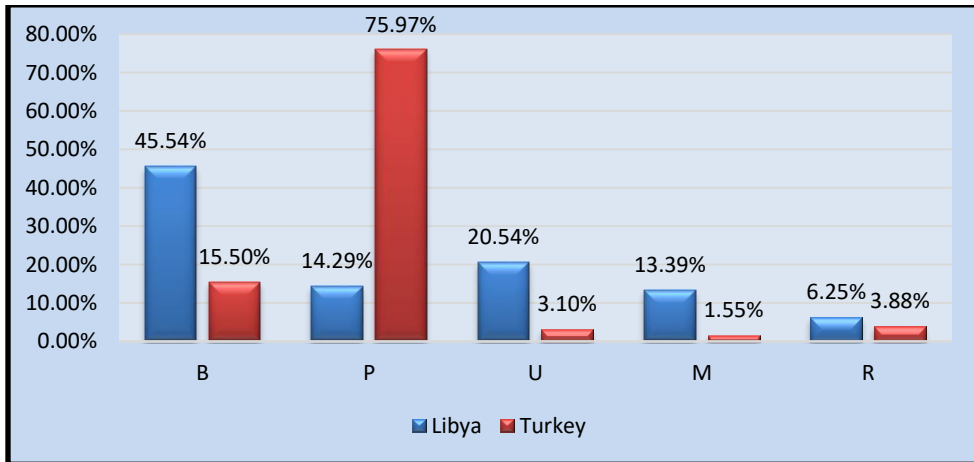


Figure 14. a Turkish students' sample answers to third problem – Part c

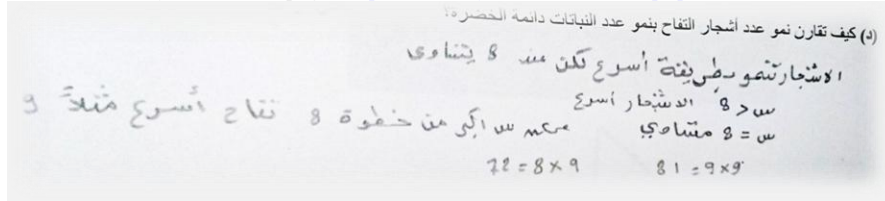
In figures 13 and 14, the students have the necessary knowledge and skills to solve part C of the third problem, and they have experience in mathematics laws. The students took the appropriate steps to achieve the solution and they analyzed the mathematical quadratic equation to find the solution. For this reason, the students' answers were accepted to be at RSL.

Problem 3 - Part d: The analysis of the answers given to the 3rd problem - part (d) according to the SOLO taxonomy.



Graph 8. The analysis of the answers given to the 3rd problem according to the SOLO taxonomy: Part d

A few students from each of the countries were able to achieve the full answer. We note that the highest percentage of students who answered this problem were Libyan students at the relational-structural level (RSL). However, this represents very low percentage, which indicates that most students have difficulty and failed in example answer and they cannot compare the growth of the number of apple trees with the growth of the number of evergreens. Therefore, students' answers could not be assessed as a product of advanced thinking. It can be concluded that the students failed to perform the transition to the advanced thinking level in problem 3 - Part d. Below are examples of answers that are accepted at RSL.



[When $x < 8$, evergreens grow faster. / When $x > 8$, apple trees grow faster. / When $x = 8$, the number of apple trees and evergreen trees is equal.]

Figure 15. a Libyan students' sample answers to third problem – Part d

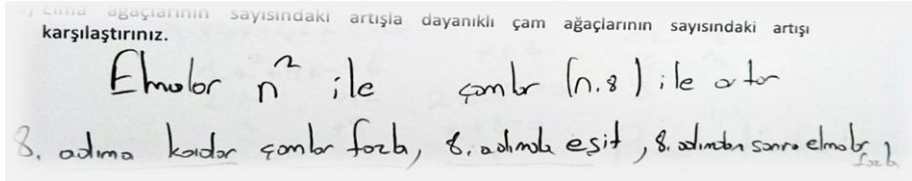
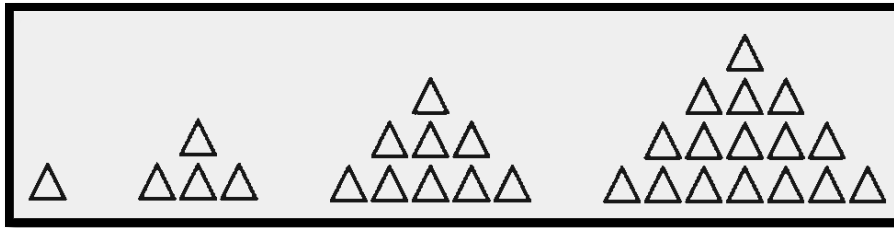


Figure 16. a Turkish students' sample answers to third problem – Part d

The answers given in figures 15 and 16 show that these two students had the necessary knowledge to find the solution and understand pattern problems. The students took the appropriate steps to achieve the solution. Students can compare how the growth of the number of apple trees with the growth of the number of evergreens. Two students answers could be assessed as a product of advanced thinking.

The results of the analysis of the students' responses to the fourth problem.

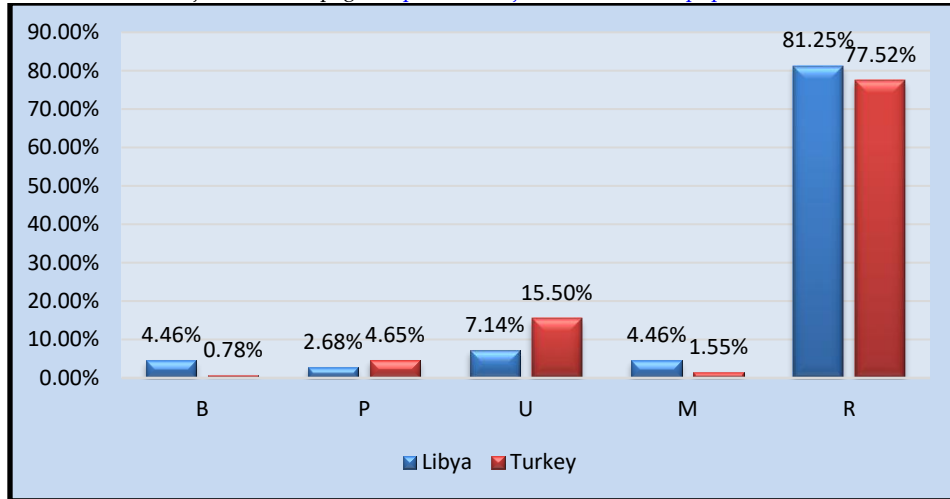
Problem 4: If we have the following diagram:



- Do you see any patterns in the diagram? If so, describe them.
- Continue the pattern for $n = 5$.
- Writing an algebraic equation of a pattern in paragraph (a).

Here are the results of the analysis of the students' responses to the four problems, which is in three parts.

Problem 4 - Part a: The analysis of the answers given to the 4th problem - part (a) according to the SOLO taxonomy.



Graph 9. The analysis of the answers given to the 4th problem according to the SOLO taxonomy: Part a

As is seen in the graph 9, it was observed that a large proportion of the Turkish and Libyan students participating in the study can succeed in advancing toward advanced thinking levels. As for the Libyan students, 81.25 % of are at the relational-structural level (RSL), and For the Turkish students, 77.52% of are at the relational-structural level (RSL). It means that students thinking levels in regarding the formulation of quadratic patterns are high and a large percentage of the Libyan and Turkish students were able to knowledge the pattern and its description. Despite this, Libyan students' level was higher than the Turkish students' level. Some of the answers given by the Libyan and Turkish students at the relational-structural level are given below:

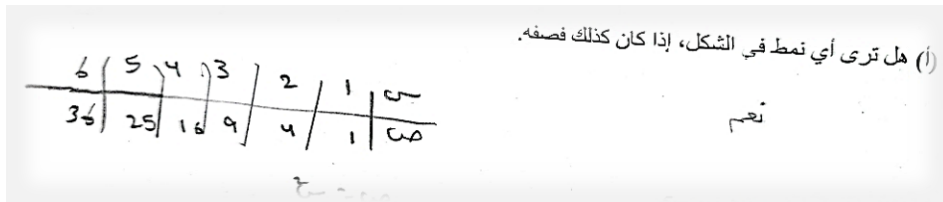


Figure 17. a Libyan students' sample answers to fourth problem – Part a

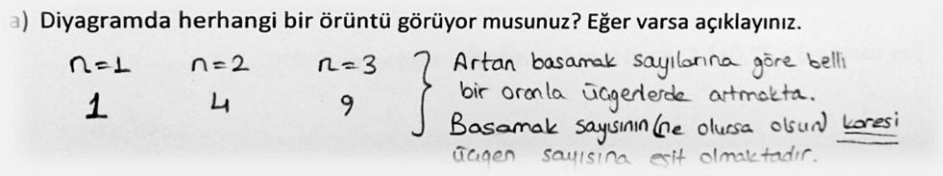
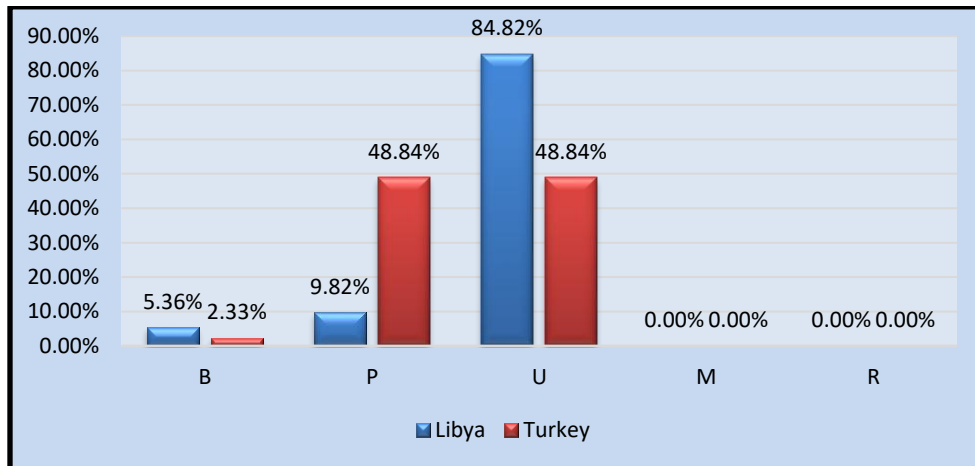


Figure 18. a Turkish students' sample answers to fourth problem – Part a

As shown in figures 17 and 18, these two students could provide a correct answer to this problem, which suggests that they had the necessary knowledge and skills to present a solution using the right method. The students took a good step for the solution, and they were able to knowledge the pattern and its description, this indicates students' understanding of quadratic patterns. For this reason, the students' answers were accepted to be at RSL.

Problem 4 - Part b: The analysis of the answers given to the 4th question - part (b) according to the SOLO taxonomy.



Graph 10. The analysis of the answers given to the 4th problem according to the SOLO taxonomy: Part b

As is seen in the Graph 10, The findings indicate that a majority of the Turkish and Libyan students participating in the study can uni-structural level thinking in solving the problem. the Libyan students, 84.82% are at the uni- structural level (USL), and 9.82% ware at the pre-structural level (PSL). For the Turkish students,48.84% are at the uni- structural level (USL), and 48.84 % ware at the pre-structural level (PSL). Some examples from solutions of students are as follows, it explains students can use arithmetic and descriptive methods properly and they can describe pattern but not generalize, meaning students cannot use algebraic methods.

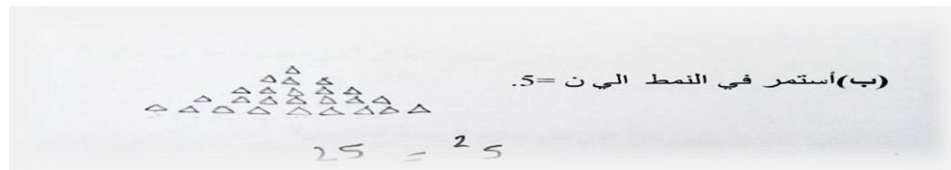


Figure 19. a Libyan students' sample answers to fourth problem – Part b

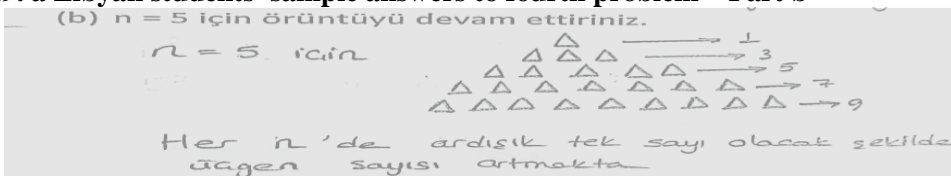
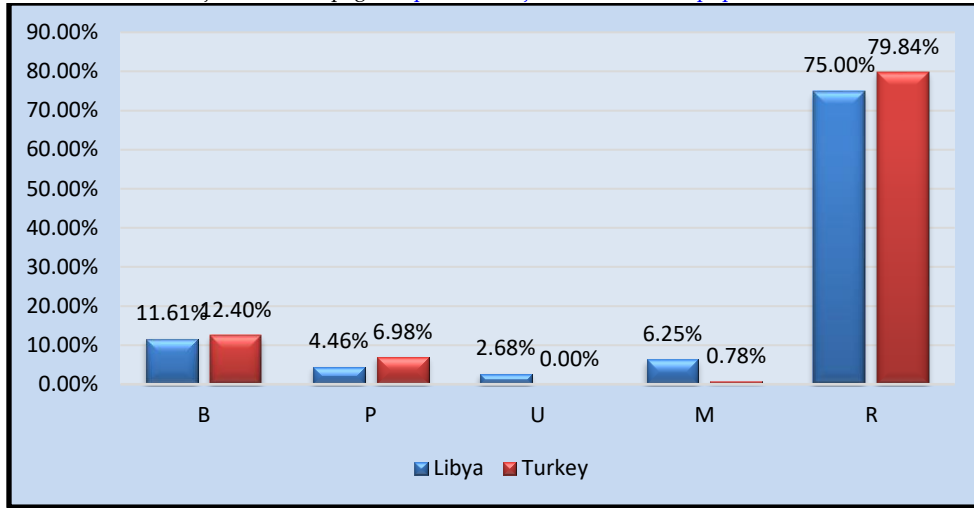


Figure 20. a Turkish students' sample answers to fourth problem – Part b

As shown in Figures 19 and 20, these two students could provide a correct answer to this problem, which suggests that they had the necessary knowledge and skills to present a solution using the right method. The students took on a good step for the solution and obviously, It explains students can use arithmetic and descriptive methods properly and they can describe pattern but not generalize. For this reason, students' answers are accepted in USL.

Problem 4 - Part c: The analysis of the answers given to the 4th problem - part (c) according to the SOLO taxonomy.



Graph 11. The analysis of the answers given to the 4th problem according to the SOLO taxonomy: Part c

As is seen in the graph 11, it was observed that a large proportion of the Turkish and Libyan students participating in the study can succeed in advancing toward advanced thinking levels. As for the Libyan students, 75.00 % of are at the relational-structural level (RSL), and For the Turkish students, 79.84 % of are at the relational-structural level (RSL). It means that students thinking levels in regarding the formulation of quadratic patterns are high and a large percentage of the Libyan and Turkish students were able to knowledge the pattern and find the formation rule of the pattern. Despite this, Turkish students' level was higher than the Libyan students' level. Some of the answers given by the Libyan and Turkish students at the relational-structural level are given below:



[student answer: $y = n^2$]

Figure 21. a Libyan students' sample answers to fourth problem – Part c

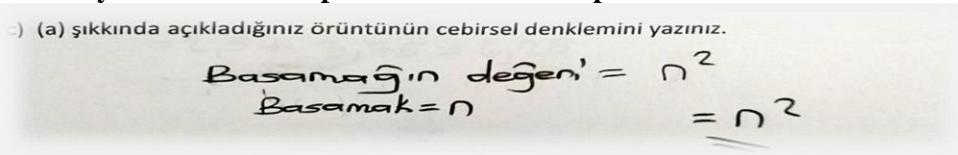
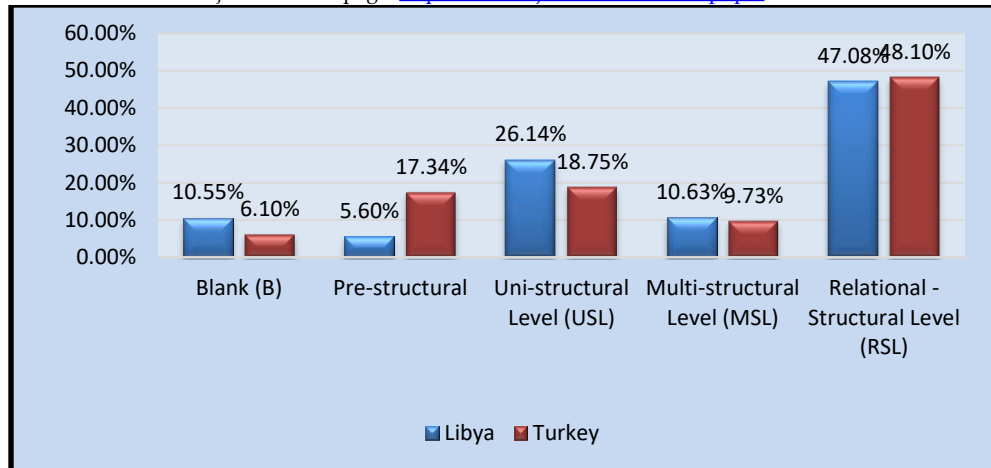


Figure 22. a Turkish students' sample answers to fourth problem – Part c

In the answers given in figures 21 and 22, where students were able to provide a correct answer. Students have the necessary knowledge and skills to present a solution using the right method. Students took a good step in the solution, they could find the formation rule of the numbers in the pattern, and they could generalize the relation and put a rule of the pattern. For this reason, the students' answers accepted to be at RSL.

Findings and Comments Related to the Second Research Question

This section presents the findings of the analysis of the responses of the Libyan and Turkish students to all four problems.



Graph 12. Overall analysis of the answers of the Libyan and Turkish students to the four problems according to the SOLO taxonomy

Graph 12 shows that 47.08% Libyan students are at the relational-structural level (RSL), 10.63% students are at the multi-structural level (MSL), 5.60% are at pre-structural, 26.14% are at uni-structural (USL) thinking levels, and the remaining are about 10.55% did not attempt to solve the problem. For the Turkish students, 48.1% of are at the relational-structural level (RSL), 9.73% students are at the multi-structural level (MSL), 17.34% are at pre-structural, 18.75 % are at uni-structural (USL) thinking levels, and the remaining 6.1% are not attempted to solve the problem.

Findings indicates that a large proportion of the Turkish and Libyan students participating in the study can succeed in advancing toward advanced thinking levels in questions requiring skill of generalization in quadratic pattern. Comparing students of the two countries, the percentage of the Turkish and Libyan students with thinking levels at MSL and RSL in the problems was somewhat similar.

In addition, Mann Whitney U - test was conducted to determine whether is there a significant difference between the levels defined by the SOLO taxonomy of the ability of Turkish and Libyan students of Generalization in Quadratic Pattern. The findings are given in table 1.

Table 1. Findings of Mann Whitney U - Test Related to Formulation Quadratic Patterns Skills

Groups	N	Mean Rank	Sum of Ranks	U	P	Z
Libya	112	118.17	13234.50	6906.500	0.555	-0.590
Turkey	129	123.46	15926.50			

According to the Mann Whitney U-Test findings, there is not a significant difference between of Turkish and Libyan students' levels in algebraic thinking test of skill of generalization in quadratic Pattern ($U = 6906.500$, $p = 0.555 > 0.05$, $z = -0.590$).

Discussion, Conclusion, Recommendations and suggestions

The current study aimed to examination the thinking levels of Libyan and Turkish students' quadratic patterns generalization according to SOLO Taxonomy, because SOLO taxonomy is used as tool to measure mathematical thinking levels of students, and SOLO taxonomy is used to evaluate the problem-solving procedures of the students and their understanding level of the concepts (Lian & Idris, 2006; Pegg & Tall, 2005) in mathematics. For this in this study, students' knowledge and

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thinking levels were analyzed according to SOLO taxonomy. SOLO taxonomy provided information about the thinking levels of the students on quadratic patterns generalization problems.

According to the findings, the study findings showed that Turkish and Libyan students have a good level in quadratic patterns generalization. It was determined that most students are the relational structure thinking level in generalization in quadratic pattern, and a large proportion of the Turkish and Libyan students participating in the study can succeed in advancing toward advanced thinking levels, where 47.1% of Libyan students were at the related structure level and 48.1% of Turkish students were at the related structure level, This means that students can integrate their knowledge and skills in strong and cohesive structure. Those students their answers were require conceptual and operational knowledge and that operational and conceptual knowledge are very essential to be successful in mathematics (Hiebert & Carpenter, 1992). In addition, enabling students to the basic rules in mathematics, especially related to quadratic pattern , their success to determine how the pattern grows, their description of the pattern, and their writing of the algebraic equation of the pattern and this is an indication of the solidity of their knowledge and skill in the formulation quadratic patterns. The results of some studies support this finding, which has shown significant progress in generalizations (Olkun, Şahin, Akkurt, Dikkartın, & Gülbağcı, 2009; Berg, 2012).

However, some students struggle to use generalization correctly and fail to arrive at conclusions in problems that require it. 10.63% of Libyan students were at the multi-structural level (MSL), 26.14% of were at uni-structural (USL) thinking levels, and 9.73% of Turkish students were at the multi-structural level (MSL), 18.75 % of were at uni-structural thinking levels (USL). but these students did not apply generalization correctly, and they did not reach a conclusion. This is attributed to some students not using an appropriate method for answering questions neglect or to oversights. Furthermore, some students used computational methods to find the steps of generalization and arrived at the conclusion and the rule formula by generalizing those steps. In a study by Lian and Idris (2006) assessing the algebraic problem-solving skills of tenth-grade students using the SOLO classification, they found that low-performing students used drawing and counting methods. Similarly, some students did not consider the number of positions in the pattern, focusing only on the difference between consecutive positions. MacGregor & Stacey (1996) observed a similar situation in their study.

At the pre-structural level, approximately 10.6% of Libyan students and 6.1% of Turkish students left questions blank. This is because some students did not attempt to answer the questions due to inattention, or because their answers were irrelevant. However, these students' results do not reflect the overall level of students' ability to formulate generalizations. According to Potter and Kustra (2012), the pre-construction level should be excluded from the levels of thinking, since students generally have no grasp of any subject at this level.

When examining the results obtained using SPSS software, no statistically significant difference ($p = 0.05$) was found between the levels of Libyan and Turkish students' ability to formulate generalizations ($U = 6906,500$, $p = 0.555 > 0.05$, $z = -0.590$). Therefore, it can be concluded that the levels of Libyan and Turkish students in generalization in quadratic patterns are equivalent.

The concept of patterns is fundamental in mathematics. understanding students' level of thinking, and their knowledge of patterns is crucial, and the methods and techniques used by teachers in the classroom play a crucial role in developing thinking skills. Therefore, the following recommendations should be considered:

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- Teachers should effectively utilize modern teaching methods to cultivate students thinking skills.
- classroom activities, questions, and worksheets should be presented in a way that effectively helps students acquire generalization to patterns.
- Students should be supported in developing their thinking patterns and improving their thinking levels through problem-solving.
- Supporting students in transferring their thinking skills from a design-based learning environment that focuses on "information retrieval" to an environment that encourages "analysis and synthesis".

Based on the study finding, the following suggestions are made:

- Numerous studies in various fields have been conducted in the educational literature according to the SOLO taxonomy. However, no studies in Libya based on this SOLO taxonomy have been found. Therefore, it should procedure many from studies according to SOLO taxonomy in different fields.
- This study was conducted on skill of quadratic patterns generalization . It is possible to conduct a study on other skills from thinking. In addition, it can work on other problems than the problems of this study.
- In studies to be made on the pattern, in future studies, students' insufficient knowledge, mistakes, and misperceptions about the concept of patterns can be determined.

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